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What is exploratory teaching in mathematics?

Anders.Sanne@ntnu.no

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In the new Norwegian mathematics curricula great emphasis is placed on exploration.

- What exactly is exploratory mathematics teaching?
- Where do these ideas come from, and how do the mathematics teachers interpret the curricula in terms of exploratory teaching?
- How is such exploratory mathematics implemented in the classroom?

At NTNU in Trondheim, we collaborate with mathematics teachers at two local schools – a junior high school and a high school. I will talk about this collaboration and show some examples of tasks and activities.

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BACKGROUND AND CURRICULUM

Landscapes of exploration

- The pupils must accept the invitation
- Fruitfull and tempting
- The tasks are not formulated/given

Skovsmose (2003)

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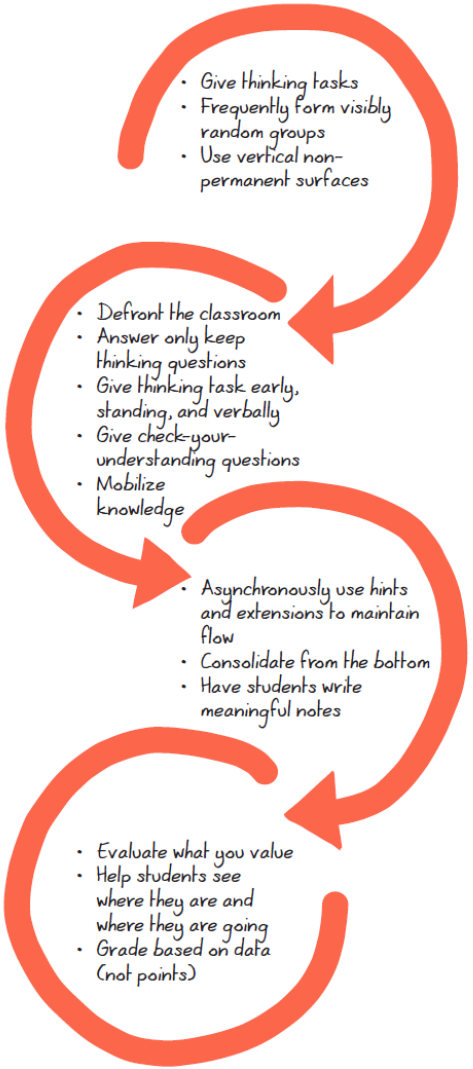
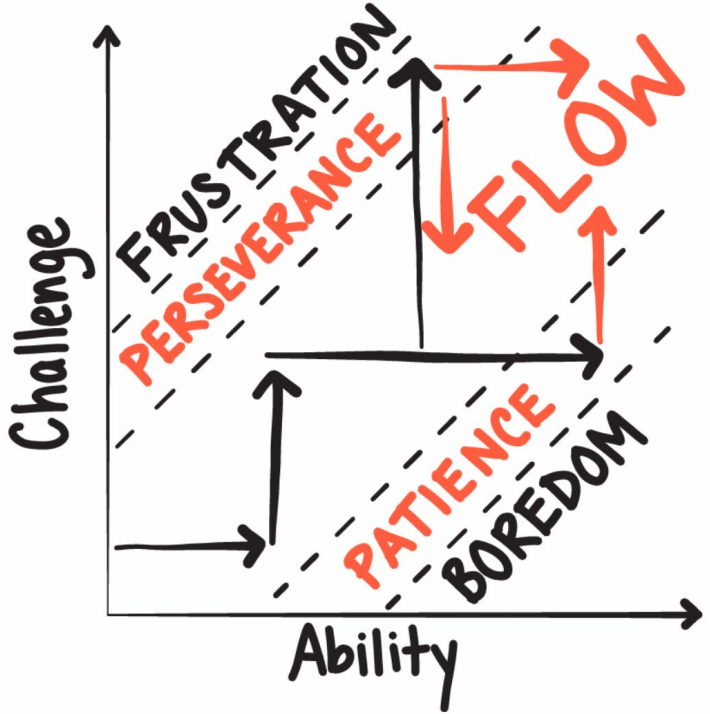
Boaler (1998)

… traditional textbook approach that emphasizes computation, rules, and procedures, at the expense of depth of understanding, is disadvantageous to students, primarily because it encourages learning that is inflexible, school-bound, and of limited use.

The Building Thinking Classrooms Framework



Liljedahl (2021, s. 281)



Exploration and problem solving

Exploration in mathematics means that the pupils search for patterns, find relationships and discuss their way to a shared understanding. The pupils shall place more emphasis on strategies and approaches than on solutions.

One of the core elements in Mathematics as stated in Norwegian National Curriculum

Exploration and problem solving

Problem solving in mathematics means that the pupils develop a method for solving a problem not previously encountered. Computational thinking is important in the process of developing strategies and approaches to solve problems, and means breaking a problem down into sub-problems that can be solved systematically. This also includes evaluating whether sub-problems can be solved best with or without digital tools. Problem solving also means analysing and reformulating known and unknown problems, solving them and evaluating whether the solutions are valid.

Explanation of the verb explore

To explore refers to experiencing and experimenting, often encouraging curiosity and sense of wonder. To explore may mean to sense, seek, discover, observe and examine. In some cases, to explore means to investigate different aspects of an issue through open and critical discussion. To explore may also mean testing or trying out and evaluating work methods, products or equipment.

Example task (exam grade 10)

Bruk påstandene ovenfor som et utgangspunkt for å vise din kompetanse innen abstraksjon og generalisering.

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Believes and Experiences

TEACHERS AND TEACHER STUDENTS

Teachers

- Pupils work individually with explorative tasks given by the theacher
- «Guided reinvention»
- Teacher do not help much
- Some kind of summary at the end of the lesson

- Difficult to engage all pupils in the given tasks
- Difficult to assess

Student teachers

- Know the ideas from the lessons at the University
- Have not seen this when they where pupils themselves
- Difficult to find good tasks and activities
- Few role models

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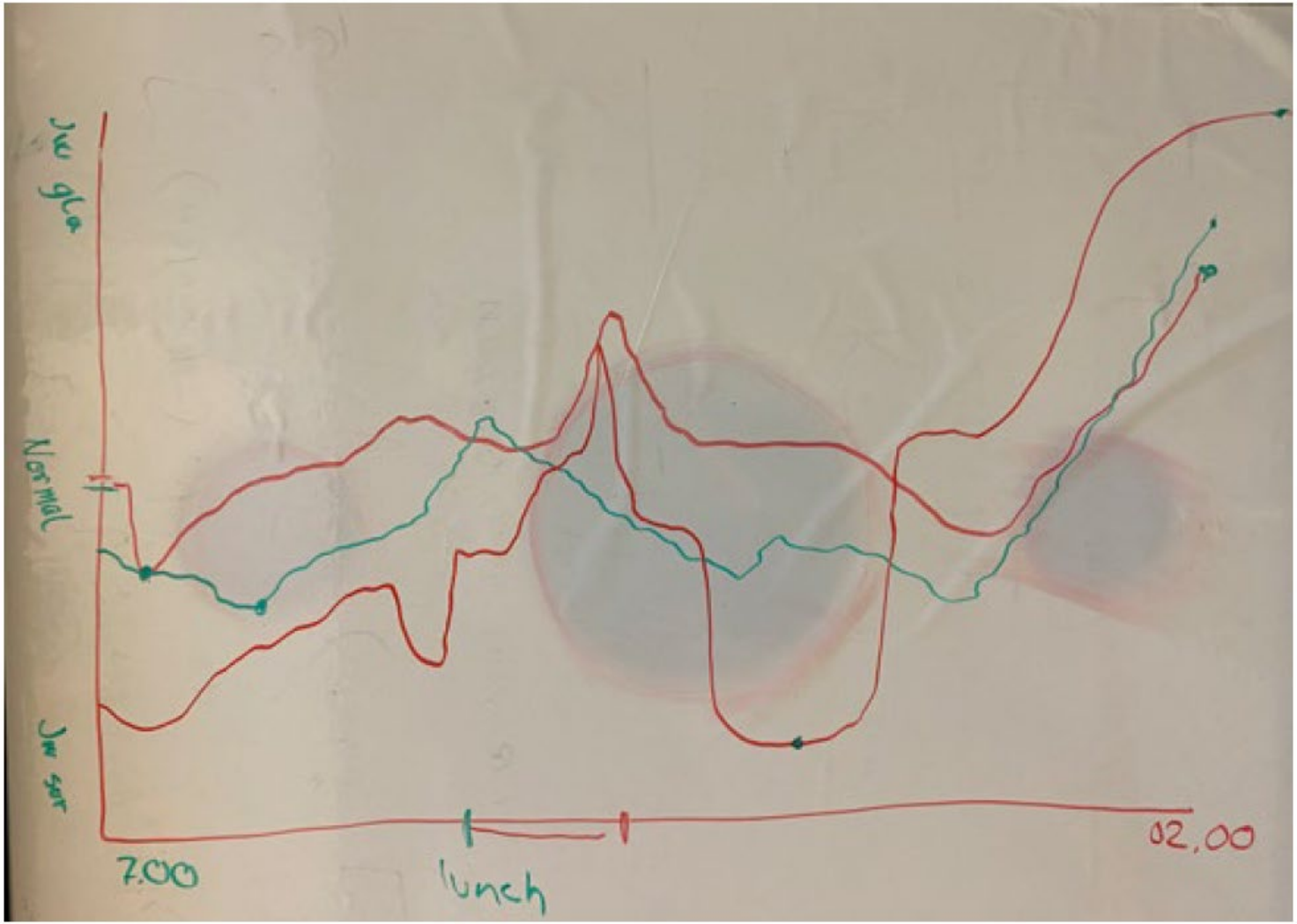
Collaboration between Teachers,
Student teachers and University teachers

EXPLORING EXPLORATIVE TEACHING IN MATHEMATICS

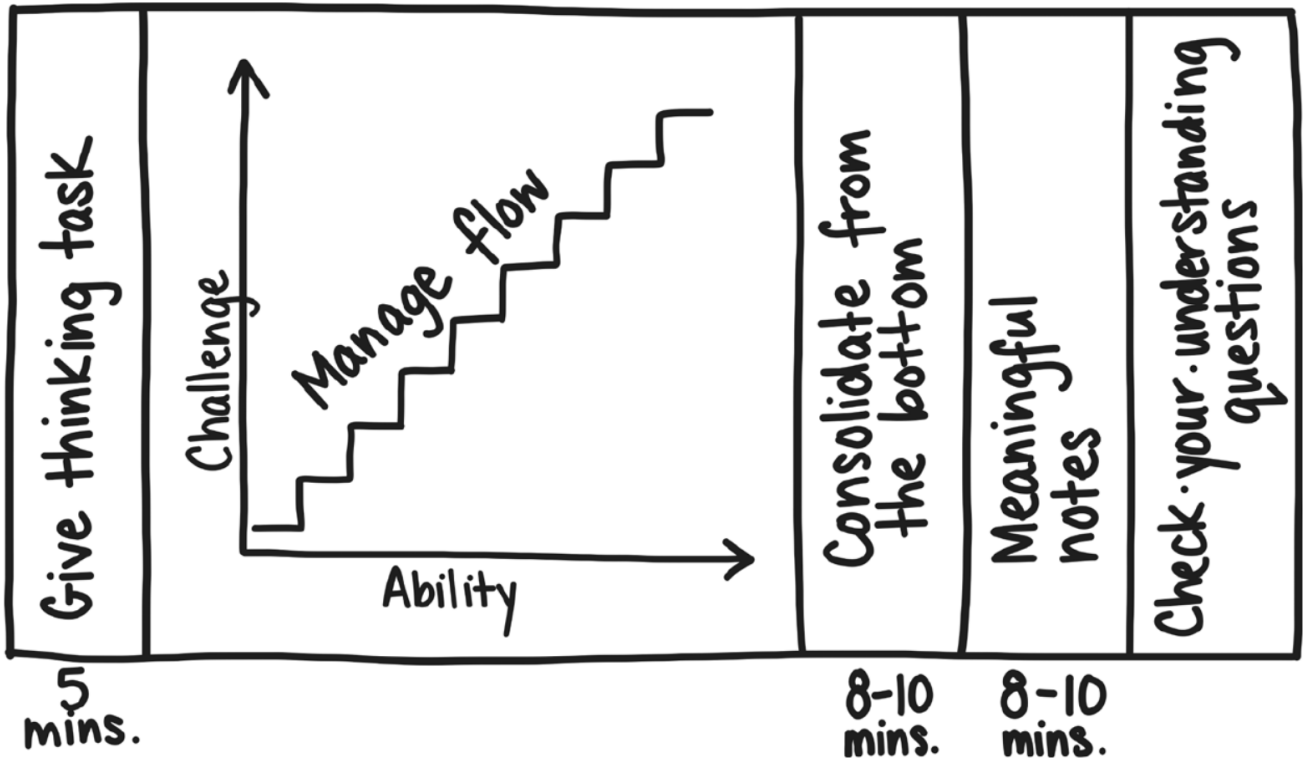
Thinking classroom

- Inspired by Liljedahl (2021)
- Working on boards
- Random grouping
- Non-curricular tasks
- Meaningful notes
- Transform “ordinary” tasks to thinking tasks

- Exploring - learning as experiencing


Normal
7.00
lunch
02.00

Typical lesson (Liljedahl 2021)



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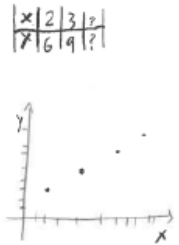
1. Hva gjorde dere på tavlene i dag?	2. Tegn skisse av det dere jobbet med.
Jobbet med verditabell og proporsjonalitet	
3. Hva gjorde de andre gruppene på tavlene?	4. Hva lærte du i dag?
Vi gjorde det samme, men kom med forskjellige tekstforklaringer på tabelene	Jeg lerte å jobbe med verditabell og mer om proporsjonalitet
5. Var det noe som var vanskelig å forstå?	6. Andre kommentarer til timene i dag?
Først var det litt vanskelig å forstå hvordan tallene hang sammen, men fant det ut senere i timen	

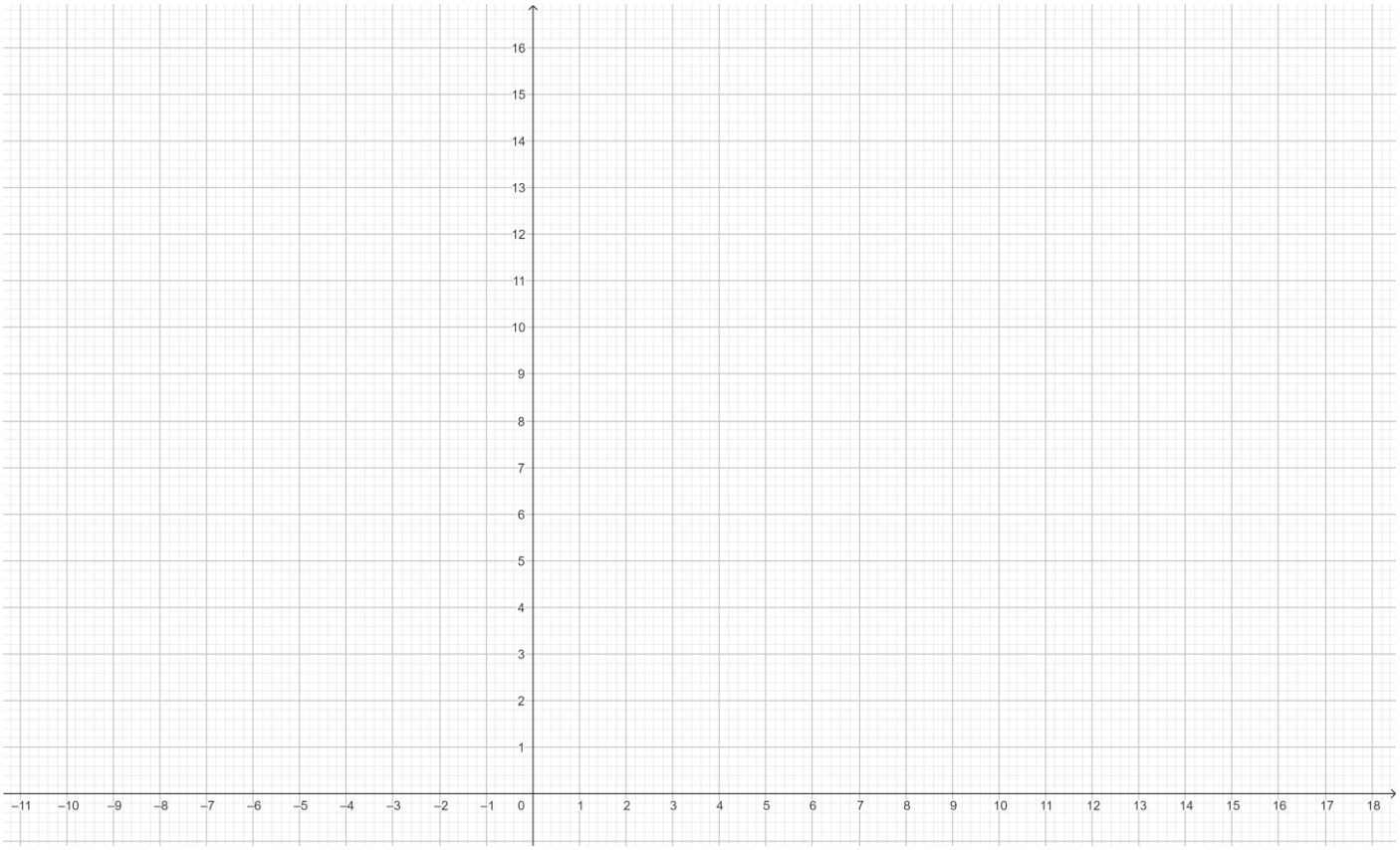
Meaningful notes

Andregradsfunksjoner

Oppgave 1

- Tegn de tre funksjonene *f*, *g* og *h* så nøyaktig du kan i koordinatsystemet nederst.
  Du trenger ikke fylle inn alle cellene i tabellen.
  Regn ut så mange punkt som du trenger for å kunne tegne en nøyaktig graf.
- Bruk blyant slik at du kan viske ut hvis du tegner feil.
- Husk at grafen til en andregradsfunksjon alltid er en parabel.
  Disse grafene er glatte – de har ingen knekkpunkt.

x	-6	-5	-4	$-\frac{7}{2}$	-3	$-\frac{5}{2}$	-2	$-\frac{3}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	2	$\frac{5}{2}$	3	$\frac{7}{2}$	4	5	6
$f(x) = x^2$																				
$g(x) = (x-3)^2$																				
$h(x) = (x+2)^2$																				



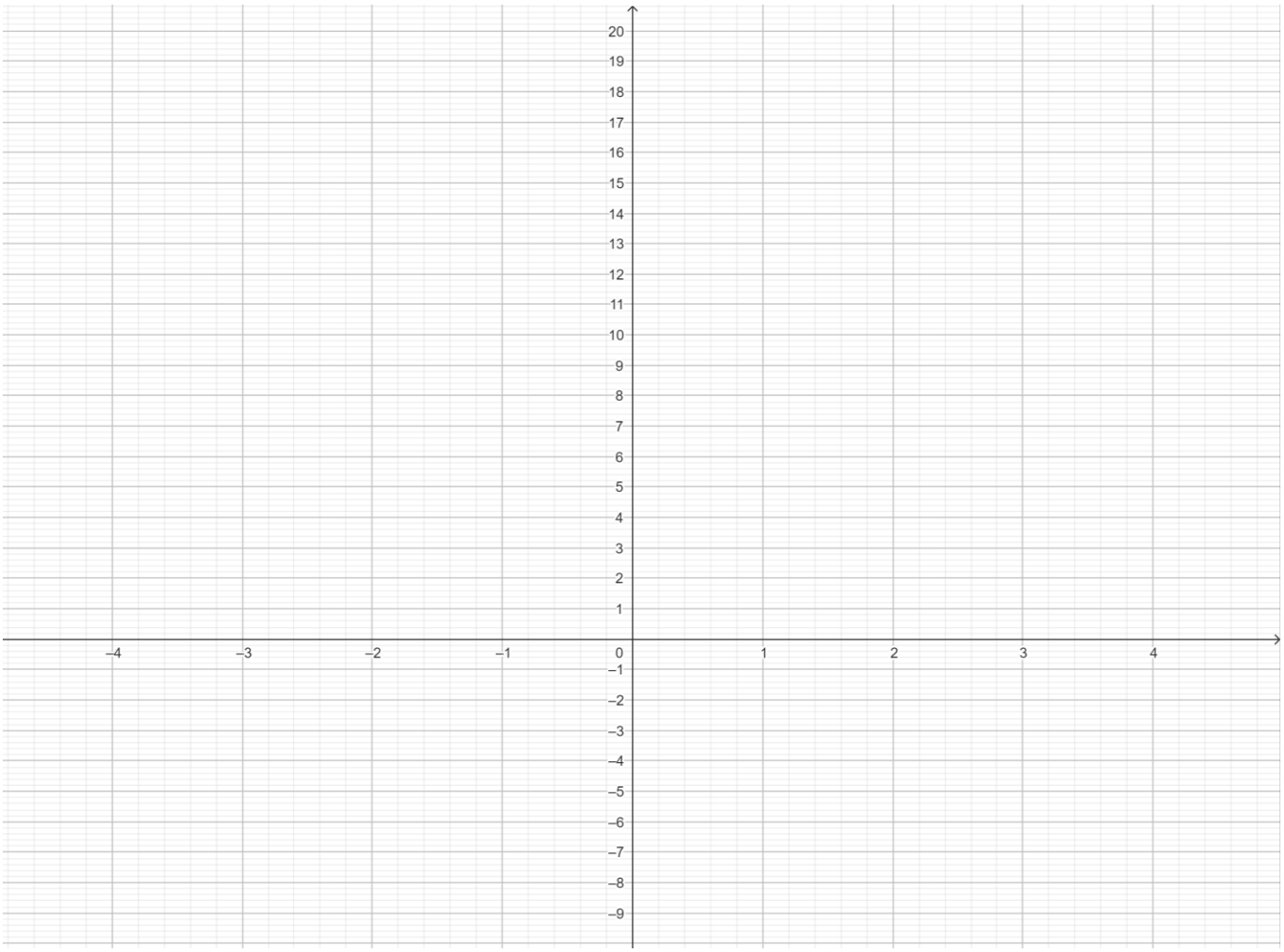
Oppgave 2

a) Tegn inn symmetrilinjene til hver av de tre grafene.
b) Marker bunnpunktet til hver av grafene.
c) Ser du noen sammenheng mellom symmetrilinje, bunnpunkt og uttrykket $(x-p)^2$?

Oppgave 3

- Tegn de tre funksjonene *k*, *m* og *n* så nøyaktig du kan i koordinatsystemet nederst.
  Du trenger ikke fylle inn alle de cellene i tabellen.
  Regn ut så mange punkt som du trenger for å kunne tegne en nøyaktig graf.
- Bruk blyant slik at du kan viske ut hvis du tegner feil.
- Husk at grafen til en andregradsfunksjon alltid er en parabel.
  Disse grafene er glatte – de har ingen knekkpunkt.

x	-3	-2	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	2	3
$k(x) = x^2$									
$m(x) = 2x^2$									
$n(x) = -x^2$									



Oppgave 4

- Funksjonene i oppgave 3 har uttrykk som kan skrives som ax^2.
  Prøv å beskrive med dine egne ord hvordan forskjellige verdier av koeffisienten *a*
  påvirker utseendet til grafen.

Some results

- Engaging for the pupils
- More fun for the teachers
- Meaning making
- Better relations between teacher and pupils
- The curriculum has some expectations, easier to argue that the teaching and learning should be explorative

Litterature

Boaler, J. (1998). Open and Closed Mathematics: Student Experiences and Understandings. *Journal for Research in Mathematics Education*, *29*(1), 41–62. http://doi.org/10.2307/749717

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Skovsmose, O. (2003). Undersøgelseslandskaber. I M. Blomhøj, H. Alrø, & O. Skovsmose (Red.), *Kan det virkelig passe? Om matematiklæring* (s. 143–158). København: Forlag Malling Beck.