Challenge students' creativity
- turn the task around and use GeoGebra

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Compare the following tasks:

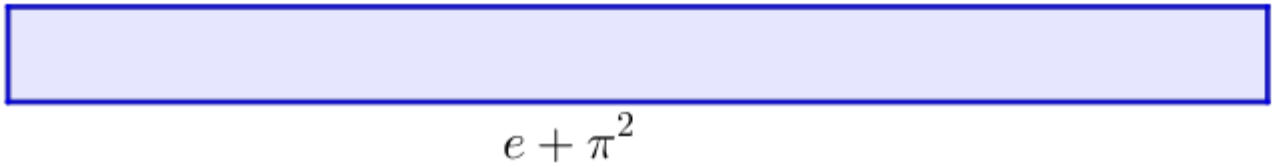
a) Determine the area of an rectangle with a length of 4 cm and a width of 3 cm.

b) Give two different examples of rectangles with the area 12 cm^2.

Give two significantly different examples of rectangles with the area 12 cm^2.

Give an example that you think that no one else in the class has provided.

Give an example that you think that no one else in the world has provided.



In mathematics education, examples play a key role

- Most often, the examples are chosen and presented by a teacher or a textbook
- Let students construct their own examples
- Students must be creative and develop solution strategies based on understanding
- Require students to provide more than one example
- Increased potential for fruitful peer discussions

Example-generating tasks

For example …

When working with *functions*, it is common to encounter tasks where a function formula is provided, and the goal is to determine various properties of the function, such as local extrema, maximum and minimum values, asymptotes, range, and more.

What happens if we instead turn the task around, by specifying some conditions and asking students to find different examples of functions that fulfil these conditions?

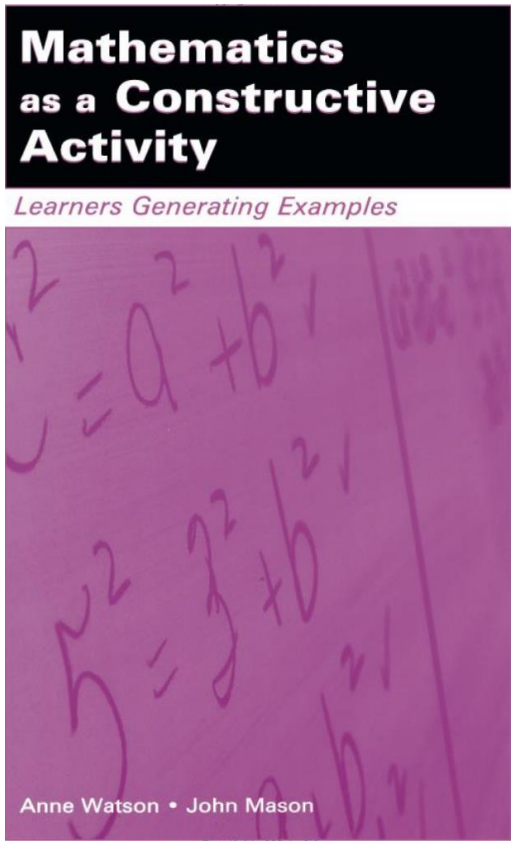
Let’s try some tasks…

www.geogebra.org/classroom/c4jrtjkh

www.geogebra.org/classroom **C4JR TJKH**

Example spaces

- Asking students to construct their own examples - a powerful pedagogical tool
- *Example spaces* - collections of examples that fulfil certain conditions
- The richness of students' example space - an indicator of their mathematical understanding



Watson, A., & Mason, J. (2005)

Two notions related to example spaces

- **Dimensions of possible variation (DofPV)**
  The features of an example that are possible to vary without losing the determining characteristics.
- **The associated ranges of permissible change (RofPCh)**
  The extent to which these dimensions can be varied.

Watson, A., & Mason, J. (2005).

Task 1

Give two different examples of quadratic functions with the minimum point (3, -2)

DofPV: the form of the function formula

RofPCh: standard form, factored form, vertex form

DofPV: scaling the graph by compressing or expanding it horizontally (e.g. changing the value of the parameter *k* in vertex form, $f(x) = k(x-3)^2 - 2$).

RofPCh: The value of *k* can be any positive real number (if written in vertex form)

Task 2

Give two different examples of trigonometric functions that have the range $-2 \leq y \leq 4$

DofPV: the form of the trigonometric function

RofPCh: sine, cosine

DofPV: the values of the parameters *A, B* or *C* in $f(x) = A\sin\big(B(x + C)\big) + D$

RofPCh: *A: 3 or -3*

B: all real numbers except 0

C: all real numbers

Task 3

Give two different examples of functions with a vertical asymptote $x = 1$

DofPV: the type of function

RofPCh: rational, logarithmic, tangent, ….

Depending on the type of function, lots of different DofPV will appear.

An example from a first-year engineering course

Give examples of two different functions, f and g, both of which have
- two vertical asymptotes, $x = -6$ and $x = 3$, as well as
- a horizontal asymptote, $y = 2$.

Note:
- Group members may have received different asymptotes.
- Check in GeoGebra if your suggested functions really have the given asymptotes.

Individual response:

$f(x) =$

$g(x) =$

Most of the students managed to provide two correct examples, in the form of rational functions.

Additional constraints to enrich students' example space

The students were asked to give two examples of **non-rational** functions that have a given horizontal asymptote, and two examples of **non-rational** functions that have a given vertical asymptote.

This additional constraint activated a further DofPV in terms of types of function, and the associated RofPCh turned out to be ample.

Given condition	**First example**	**Second example**
Horizontal asymptote $y = 2$	$f(x) = \frac{1}{\ln x} + 2$	$g(x) = \arctan(x) + \left(2 - \frac{\pi}{2}\right)$
Horizontal asymptote $y = 2$	$f(x) = 2 - 2^x$	$g(x) = 2^{-x} + 2$
Vertical asymptote $x = 2$	$f(x) = \frac{1}{e^{x-2} - 1}$	$g(x) = \tan\left(\frac{\pi x}{4}\right)$
Vertical asymptote $x = 2$	$f(x) = \frac{1}{2^x - 4}$	$g(x) = -\frac{1}{2^x - 4}$

To summarize

- One way to get students to be creative and increase the chance that their example space is enriched is to ask for several examples that are as different as possible. This will likely lead to reflections on *ranges of permissible change*.
- Another way is to add additional constraints that render "typical examples" invalid. This can prompt students to think in new ways and discover new *dimensions of possible variation*.

References

Fahlgren, M., & Brunström, M. (2023). Designing example-generating tasks for a technology-rich mathematical environment. *International Journal of Mathematical Education in Science and Technology*. DOI: 10.1080/0020739X.2023.2255188

Watson, A., & Mason, J. (2005). *Mathematics as a constructive activity: Learners generating examples*. Routledge.

Many thanks for your attention!